ABSTRACTS OF ARTICLES DEPOSITED AT VINITI *

ABILITY OF SOLUTIONS OF CARBOXYLIC ACID SALTS IN THE QUASISOLID STATE TO LOWER THE HYDRODYNAMIC RESISTANCE OF WATER

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UDC 532.517.4:661.185

At present the hydrodynamic resistance of a liquid is lowered by adding high-molecular-weight compounds and micelle-forming surface-active materials (SAM) [1].

We have investigated the physicochemical properties of solutions of salts of saturated carboxylic acids, and have determined parameters which we proposed to use to predict the ability of these salts to lower hydrodynamic resistance in the turbulent flow of a liquid. We investigated sodium and potassium salts of myristic, pentadecanoic, palmitic, and stearic acids, and soap chip (MRTU 18/233-68), which is a mixture of sodium salts of carboxylic acids.

The activity coefficients of the counter ions and the viscosity of the SAM in question were measured. Their Krafft points and the effect of various additives on them were determined. The temperature dependence of the magnitude of the decrease in hydrodynamic resistance was investigated. The decrease in friction losses for solutions of individual salts was observed in the 3-8°C range and near 30°C for a mixture (soap chip).

The results obtained for the dependence of the activity coefficients of counter ions, viscosity, and solubilization power on temperature and the SAM concentration and the effect of lowering the hydrodynamic resistance lead to the assumption that there is present in solutions of salts of saturated carboxylic acids the so-called quasisolid state characterized by the presence of nonspherical micelles and two Krafft points T_k' and T_k corresponding to the transition of the solution from the solid to the quasisolid state T_k' and then to liquid form. One of the distinguishing characteristics of a solution in the quasisolid state is its ability to lower the hydrodynamic resistance of a liquid within a definite temperature range. The upper limit of this temperature range is the T_k of the given compounds or mixture of them.

In view of this, we investigated the effect on the Krafft point of adding an electrolyte (Na_2CO_3) and propyl alcohol. The addition of the electrolyte increased T_k, while propyl alcohol decreased it (Fig. 1). The additives investigated have a similar effect on the temperature range in which the hydrodynamic resistance is lowered. The addition of the electrolyte displaced this range toward higher temperatures and broadened it, while the addition of propanol displaced it toward lower temperatures and narrowed it.

Thus, by using the literature values [2] of the Krafft points an SAM additive can be chosen to lower the hydrodynamic resistance and, within certain limits, to control the temperature range in which the lowering occurs.

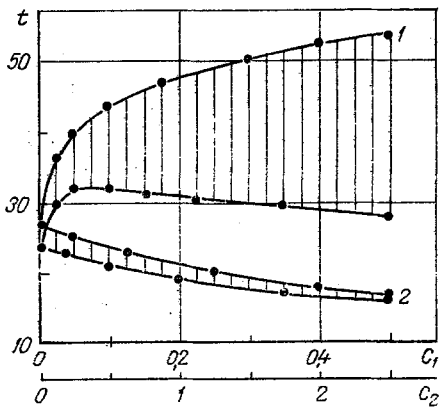

Fig. 1. Effect of concentration of 1) sodium carbonate, C_1, moles/liter, and 2) propanol C_2, %, added to a solution of sodium myristate on the Krafft point and the size of the temperature range within which the hydrodynamic resistance is lower (shaded region).

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 37, No. 4, pp. 744-753, October, 1979.

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Reg. No. 2101-79 Dep.
Original article submitted Dec. 6, 1978.
Abstract submitted May 14, 1979.

EXPERIMENTAL STUDY OF HIGH-CONCENTRATION DISPERSED FLOWS IN A HORIZONTAL CHANNEL OF CIRCULAR CROSS SECTION

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UDC 536.244

We have investigated the flow of a dispersed medium (air–boron carbide particles with an average diameter $d_s = 4.85$ μm) for a volume concentration of the solid component between 0.117 and 0.215, which corresponds to a change in the flow rate ratio (the ratio of the flow rates of the solid and gaseous components) from 228 to 479. It was shown that in the range of volume concentration investigated the main contribution to the hydraulic resistance in the stabilized portion comes from a quantity related to the interaction of the solid component with the channel wall. Using the conventional representation of the interaction of a dispersed flow with a wall as a process of "dry" friction, the coefficient of friction of a dispersed material can be introduced in a way analogous to that used in processing results for a single-phase fluid [1, 2]:

$$\Delta P_s = \lambda_{fr} L/D \frac{\rho_{vol} v_s^2}{2}, \quad (1)$$

where ΔP_s is the pressure loss resulting from the motion of the solid component; λ_{fr}, coefficient of friction of the dispersed flow; L and D, channel length and diameter, respectively; $\rho_{vol} = \beta\rho_s$, volume density of the stream; β, volume concentration of the solid component; ρ_s, density of the material of the solid component; $v_s = G_s/\beta\rho_s F$, velocity of the solid component; F, cross-sectional area of the channel; and G_s, mass flow rate of the solid component.

Processing the experimental data by the method of least squares leads to the following dimensionless relation:

$$\lambda_{fr} = 459 Fr_s^{0.79}, \quad (2)$$

where $Fr_s = gd_s/v_s^2$ is the Froude number. The value of the Froude number varied from $1.9 \cdot 10^{-6}$ to $5.65 \cdot 10^{-6}$.

By using the longitudinal distribution of the volume concentration obtained by the cut-off method the longitudinal profile of the slip ratio (the ratio of the velocities of the components) was determined. The procedure is described and the results of an experimental determination of the critical flow velocity at which there is fallout of particles of the solid component in the lower part of the channel are presented. For volume concentrations from 0.117 to 0.235 the critical flow velocity varied from 1.91 to 3.13 m/sec.

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Reg. No. 2096-79 Dep.
Original article submitted Jan. 12, 1979.
Abstract submitted Apr. 10, 1979.

EVOLUTION OF A VORTEX IN A DILATANT FLUID

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UDC 532.516

We consider the problem of the diffusion of a two-dimensional vortex in an incompressible non-Newtonian power-law fluid. We assume that at zero time a rotational motion develops in a dilatant fluid close to the origin of coordinates. Such a delta-function velocity distribution corresponds to a finite total angular momentum of the fluid.

The azimuthal velocity of the fluid in the evolution of such a two-dimensional vortex is found by solving the quasilinear parabolic equation

$$\frac{\partial v}{\partial t} = \frac{\nu}{r^2}\frac{\partial}{\partial r}\left[r^2\left|\frac{\partial v}{\partial r} - \frac{v}{r}\right|^{n-1}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)\right], \quad n > 1. \tag{1}$$

The law of conservation of angular momentum requires the solution of Eq. (1) at any instant to satisfy the integral condition

$$\int_0^\infty v(r, t)\, r^2 dr = l = \text{const}. \tag{2}$$

The exact self-similar solution of problem (1), (2) is

$$v(r, t) = \begin{cases} \dfrac{4lr}{\tau^\alpha}\left[2(n-1)\left(\xi_0^{\frac{1}{2n}} - \xi^{\frac{1}{2n}}\right)\right]^{\frac{n}{n-1}}, & \xi < \xi_0, \\ 0, & \xi \geqslant \xi_0. \end{cases} \tag{3}$$

where

$$\tau = Tt; \quad T = \frac{4^{2n}\nu l^{n-1}}{\alpha}; \quad \xi = \frac{r^4}{\tau^\alpha}; \quad \alpha = \frac{2}{2n-1} > 0; \quad \xi_0 = [2(n-1)]^{\frac{2n}{1-2n}}\left\{2nB\left(2n, \frac{2n-1}{n-1}\right)\right\}^{-\frac{2(1-n)}{2n-1}}.$$

The solution (3) is finite at any instant in the spatial variable. Physically this means that a shear perturbation is propagated in a dilatant fluid in the form of a shear wave whose front moves through the medium with a finite velocity. Therefore the perturbed region where $v > 0$ has a finite size at any instant. The velocity and tangential stresses are continuous at the wave front.

The position of the wave front is determined from the condition $\xi = \xi_0$. Analysis shows that the velocity of the wave front decreases with time, but the perturbation can penetrate indefinitely far into the medium.

In the limit $n \to 1$, which corresponds to considering the diffusion of a vortex in a Newtonian fluid, the velocity of propagation of shear perturbations increases indefinitely.

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Reg. No. 2103-79 Dep.
Original article submitted Mar. 20, 1978.
Abstract submitted May 3, 1979.

MODEL OF JET FLOW OF A GAS IN A DEEP LAYER OF LIQUID

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UDC 536.532.2

The interaction of a liquid with a gas jet which discharges into it is one of the fundamental links in a chain of processes which occur in chemical reactors, converters, and Martin furnaces. Among the large number of problems related to these processes a very important one is the study of the discharge of a gas jet vertically upward. As the simplest flow scheme we consider the jet flow of immiscible fluids. The solution is constructed by matching asymptotic expansions within the framework of the boundary layer model under the assumption that the boundaries of the jet are sharp smooth lines on which the conditions of continuity of velocities, temperatures, and tangential and thermal stresses must be satisfied.

The solution leads to the following expressions for the half-thickness of a gas jet:

$$\frac{y_*}{a_0} = 1 + 1.72\varkappa^{-1/2}\,\mathrm{Re}^{-1/2}\,\beta^{1/2} + (1-A)\,\beta + \varkappa^{-1/2}\,\mathrm{Re}^{-1/2}\,(1.29-6.20)\,\beta^{3/2}, \qquad (1)$$

where y_* is the jet boundary and a_0 is the half width of the nozzle;

$$A = \left(\frac{\rho_2}{\rho_0} - 1\right)\frac{P_0}{\rho_2 U^2}\;; \quad \beta = \frac{\rho_2 g}{P_0}\,\xi;\quad \varkappa = \frac{\rho_2 g a_0}{P_0}\;;\quad \mathrm{Re} = \frac{\rho_0 U_0 a_0}{\mu_0}\;;\quad \xi = \frac{x}{a_0}\;.$$

It is clear from (1) that with the approximations which were made in the calculations the jet boundaries diverge for sufficiently large velocities and converge with a decrease in gas velocity. This shows that for certain values of the velocity jet flow through the whole depth of the liquid becomes impossible.

In this case it is necessary to consider another flow model, viz., bubbly flow. In this way the limit of applicability of the flow model considered was established.

NOTATION

T, temperature; ρ, density; P, pressure; μ, viscosity; U, velocity; x, longitudinal coordinate. Subscript 0 refers to gas parameters at nozzle exit, and 2 to liquid parameters.

Reg. No. 2097-79 Dep.
Original article submitted Dec. 20, 1977.
Abstract submitted Apr. 19, 1979.

LUBRICATION OF ELASTIC CYLINDERS

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UDC 532.516.5

In the problem of the rolling and sliding of elastic cylinders separated by a film of viscous fluid, the thickness of the film h(x) is given by the expression

$$h(x) = h(c) + f(x) - f(c) + \frac{4}{\pi E'}\int_a^c [K(t, x) - K(t, c)]\,p(t)\,dt, \qquad (1)$$

where a and c are the boundaries of the film; p, pressure; E', reduced elastic modulus of the cylinders; K(t, x), Green's function for elastic displacements; and f(x) characterizes the shape of the cylinders. The elastic hydrodynamic problem was investigated for the contact of a rigid cylinder with an elastic layer of thickness l, and for an elastic cylinder of radius R between rigid plates. For $a \ll l$ and $a \ll$ R a Green's function is obtained for a half plane. If h(c) $\ll$ R the decrease δ in the diameter of a cylinder is given by the expression

$$\delta = (4q/\pi E_2')\,(\ln 2Rb^{-1} + 0.18),$$

where E_2' is the reduced elastic modulus of the cylinder; q, load per unit length of the cylinder; and b, Hertz half width of the contact region. For the average film thickness h_0 the formula

$$h_0/R = 1.75\,(\mu_0\alpha u/R)^{3/4}\,(p_0/E')^{-1/4}$$

holds, where μ_0 and α are, respectively, the viscosity and pressure coefficient of lubrication, u is the velocity of rolling, and p_0 is the Hertz pressure.

Taking account of the effect of tangential stresses over the thickness of the film and the kinematics of the surfaces leads to the equations

$$\frac{d}{dx}\left[h^3\exp(-Lp)\frac{dp}{dx}\right]=\frac{d}{dx}\left\{\left[1-FV^2\int_{-\infty}^{\infty}\frac{\tau dt}{t-x}-\frac{MN}{2\pi}\int_{-\infty}^{\infty}\frac{h}{t-x}\frac{dp}{dt}\,dt+\frac{1-2\nu}{1-\nu}[Np]\right]h\right\},$$

$$h=1+x^2-c^2+D\int_a^c p(t)\ln\frac{c-t}{|t-x|}+\frac{1-2\nu}{1-\nu}N\int_c^x h\frac{dp}{dt}\,dt, \qquad (2)$$

$$p(a)=p(c)=-\frac{dp(c)}{dx}=0,$$

$$\tau+\frac{\exp[L(p-1)]}{h}\left[-F\int_{-\infty}^{\infty}\frac{\tau dt}{t-x}+\frac{MN}{2\pi}\int_{-\infty}^{\infty}\frac{h}{t-x}\frac{dp}{dt}\,dt-\frac{1-2\nu}{1-\nu}Np\right]=\frac{\exp[L(p-1)]}{h}, \qquad (3)$$

where $L = \alpha p^*$, $p^* = 6\mu_0(u_1+u_2)\sqrt{2Rh(c)}/h^2(c)$; u_1 and u_2, velocities of the surfaces; $F = (2/\pi E'h_0)\mu_0\exp L(u_1+u_2)$; $V = (u_2-u_1)/(u_1+u_2)$; τ, average value of the tangential stress over the thickness of the film relative to $\mu(p^*)(u_2-u_1)/h(c)$; $\mu(p) = \mu_0\exp(Lp)$; p, relative to p^*; x, a, c, and t, relative to $\sqrt{2Rh(c)}$; $M = \sqrt{2h(c)/R}$; and $N = p^*/E'$. It is clear that problems (2) and (3) are separated for $FV^2 \ll 1$. For a high velocity of rolling the tangential stresses in (3) change the kinematics of the surfaces significantly. It follows from (2) that, for $L \gtrsim 5$, $h \approx 1$ independently of the shape of the cylinders. Therefore, to lubricate heavily loaded bodies fluids with a strong dependence of viscosity on pressure are recommended. This property of a lubricant leads to a smoothing out of the irregularities in contact.

Reg. No. 2095-79 Dep.
Original article submitted Oct. 10, 1978.
Abstract submitted Apr. 27, 1979.

NUMERICAL CALCULATION OF UNSTEADY MOTION OF A LIQUID IN A CHANNEL

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UDC 621.822

An algorithm is described for the numerical calculation of the dynamics of a liquid in a channel with a valve at the end by using the method of characteristics. The unsteady motion of a viscous slightly compressible liquid in a long channel is described by the dimensionless equations

$$-\frac{\partial p}{\partial x}=\frac{\partial v}{\partial\tau}+\psi v+\varepsilon,\quad -\frac{\partial p}{\partial\tau}=\frac{\partial v}{\partial x}, \qquad (1)$$

where

$$\varepsilon=\frac{\psi}{2}\int_0^{\tau}\frac{\partial v}{\partial\tau_1}W(\tau-\tau_1)\,d\tau_1;\quad \psi=\frac{8\nu}{r^2}\frac{L}{c}; \qquad (2)$$

$$W(\tau)=\sum_{n=1}^{\infty}\exp\left(-\frac{\psi z_n^2}{8}\tau\right);\quad p=\frac{P}{\rho cV_0};\quad v=\frac{V}{V_0};\quad \tau=\frac{ct}{L};\quad x=\frac{X}{L}.$$

To simplify the algorithm for the numerical solution of Eqs. (1) the integral (2) was approximated by a sum of exponential functions [1], and the approximation for $\psi\tau \leqslant 10^{-4}$ was improved by increasing the number of exponential functions to five. The flow parameters at the end of the channel were calculated by solving simultaneously a finite-difference equation on a straight characteristic and the equation for the flow rate through the valve

$$p_j = p_{j-1} - v_j + v_{j-1}(1 - \psi\Delta x) - \varepsilon_{j-1}\Delta x;$$
$$v_j = \alpha\sqrt{|p_j - p_c|}\,\mathrm{sign}\,(p_j - p_c), \qquad (3)$$

where the subscript j refers to values of the flow parameters at the end of the interval Δx, and $j - 1$ to the beginning of the interval. It was established that system (3) does not have to be solved by an iterative method [2], but can be obtained in the explicit form:

$$v_j = 0.5\alpha\left(-\alpha + \sqrt{\alpha^2 + 4|\beta|}\right)\mathrm{sign}\,\beta; \quad p_j = \beta - v_j + p_c,$$

where $\beta = p_{j-1} + v_{j-1}(1 - \psi\Delta x) - \varepsilon_{j-1}\Delta x$.

The calculated frequency characteristics of a channel with a valve were confirmed by the experimental data in [3]. It was shown that for $\psi \leqslant 0.001$ channel friction can be neglected, and for large ψ the rearrangement of the velocity profile (2) has a definite effect on the dynamics.

NOTATION

X, axial coordinate; t, running time; V(X, t), axial velocity averaged over cross section; P(X, t), pressure in cross section of channel; r, L, radius and length of channel; ν, ρ, kinematic viscosity and density of liquid; c, speed of sound in liquid; z_n, roots of second-order Bessel function of the first kind; V_0, arbitrary reference value of velocity; α, dimensionless conductance of valve; p_c, dimensionless discharge pressure beyond valve.

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Reg. No. 2102-79 Dep.
Original article submitted Jan. 2, 1979.
Abstract submitted May 11, 1979.

NUMERICAL MODELING OF SOLUTIONS OF SOLIDIFICATION PROBLEMS

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UDC 536.12:536.248.2

A mathematical model of solidifcation (melting) problems is generally a mathematical model of a nonlinear unsteady heat-conducting problem with nonlinearities of the first, second, and third kinds, i.e., a general nonlinear problem [1]. In such problems a body generally passes through the whole range of temperatures from cryogenic to the melting point, and the thermophysical characteristics of the materials (nonlinearities of the first kind), heat fluxes at boundaries (nonlinearities of the second kind), and internal heat sources and sinks (nonlinearities of the third kind) vary appreciably with temperature [1]. In solidification problems heats of internal transformations L are liberated in a temperature range $\Delta T_L = T_\ell - T_s \neq 0$. In Stefan problems, which are a special case of solidification problems, it is assumed that L is not liberated in a two-phase zone ΔT_L, but on a surface having a single (constant or variable) temperature T_L ($\Delta T_L = 0$). In solving solidification problems by numerical methods using analog, digital, or hybrid computers, Stefan problems are solved as solidification problems; i.e., it is explicitly or implicitly assumed that L is liberated in a certain temperature range $\Delta T_L \neq 0$. The accuracy, time, and cost of solving Stefan problems by numerical methods as compared with analytic solutions depend strongly on that temperature range $\Delta T_L \neq 0$ which replaced the condition $\Delta T_L = 0$.

An analysis is given of the numerical modeling of solutions of solidification problems by various methods [2-4] and also by modernized methods. This investigation enabled us to explain the effect of so-called methodological factors. Characteristics of modernizations performed consist in special methods for taking account of heats of internal transformations

$L=\sum_{i=1}^{n} L_i$ in the temperature range $\Delta T_L=\sum_{i=1}^{n}\Delta T_{Li}$, and in the use of "moving" modes (R_{qv} and R_{q_v} with R_τ). An analysis of the effect of thermophysical factors is given: a) values of L; b) $\Delta T_L(\Delta T_L = 0,\ \Delta T_L \neq 0,\ \Delta T_{L1} < \Delta T_{L2} < \Delta T_{L3})$; c) λ (partial linearization); d) T_L, T_ζ, T_s; and computational factors: a) the size of the space-net (h) and time intervals ($\delta\tau$); b) h and $\delta\tau$ as functions of the position of the two-phase zone (reduction of h in the region of the two-phase zone); c) schemes of arranging nodes and elementary volumes (Π, T, Γ schemes); d) oscillations of solutions in stable difference schemes; e) range of "smearing out" L over ΔT_L; f) schemes for taking account of nonlinearities (iterative, noniterative, mixed) etc.

It is shown that numerical modeling of solutions of solidification (melting) problems can be successfully performed with adequate accuracy on any types of computers (AVM, TsVM, GVM). It must be treated as a multifactor experiment on models having a physical nature different from the thermal phenomenon being studied. The procedures for planning and carrying out these experiments must be similar to those employed in the study of large systems.

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Reg. No. 2094-79 Dep.
Original article submitted June 6, 1979.
Abstract submitted May 8, 1979.

REFINED NUMERICAL CALCULATION OF RERADIATION AND SCATTERING IN RADIATIVE HEAT-TRANSFER PROCESSES

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UDC 536.3

We consider problems of calculating resolving optical-geometrical radiation characteristics and monitoring the error of the relative values of the reradiated and scattered energy for both optically uniform and optically nonuniform volume and surface zones. On the basis of an analysis of the integral equations of radiative heat transfer written for the reradiated and scattered parts of the radiation in the reduced resolving radiation coefficients f_{ij}, an approximating system of linear algebraic equations was obtained ensuring the required accuracy in determining the reradiated and scattered parts of the radiation. This is particularly important for radiating systems having a small number of zones and strongly reflecting surfaces and scattering components in the volume zones:

$$f_{ij}=\psi_{ij}\cdot B_j+\sum_{p=1}^{n}\sum_{h=1}^{n'} R_h\psi_{ih}f_{hj}+\sum_{k=1}^{m}\sum_{g=1}^{m'}\left(\frac{\alpha}{\alpha+\beta}\right)_g\psi_{ig}f_{gj}$$

where

$$(i,\ j=1,\ 2,\ \ldots,\ m+n;\ \ F_h\in F_p;\ \ V_g\in V_k),$$

$$B_j=\begin{cases}\dfrac{\alpha_j}{\alpha_j+\beta_j} & \text{for } j=1,\ 2,\ \ldots,\ m;\\ A_j & \text{for } j=m+1,\ m+2,\ \ldots,\ m+n;\end{cases}$$

α and β are, respectively, the absorption and scattering coefficients of the medium, m^{-1}; ψ_{ij}, generalized angular coefficient of radiation from zone i to zone j; F_p, area of surface zone p, m^2; V_k, volume of volume zone k, m^3; m and n, numbers of volume and surface zones in

the system; m' and n', numbers of domains of integration in the volume and surface zones, respectively; R_h, reflectivity of zone p in domain of integration h; A_j, absorptivity of surface zone j.

A system of linear algebraic equations is presented enabling a direct calculation of the reradiation and scattering in multizone systems to be performed using triangular matrices. This greatly increases the possibilities of solving radiative heat-transfer problems on modern digital computers. Closure and reciprocity relations are presented.

The proposed algorithm for refined calculations was tested, and the error in determining the reduced resolving angular radiation coefficients was estimated.

Reg. No. 2099-79 Dep.
Original article submitted Feb. 13, 1979.
Abstract submitted Apr. 25, 1979.

TEMPERATURE DISTRIBUTION IN A FINITE CYLINDER HEATED BY RADIATION PULSES

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UDC 536.24.02

Knowledge of the temperature distribution in the receiving element of a pulsed electromagnetic radiation heat detector makes it possible to determine its zonal characteristic resulting from the nonequivalent effects of radiation on various points of the surface of the receiving element.

We have found the temperature distribution in a receiving element having the form of a finite isotropic cylinder for convective heat transfer at its boundaries by solving the heat-conduction equation in cylindrical coordinates (r, φ, z)

$$a_0^2\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial T}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2 T}{\partial \varphi^2}+\frac{\partial^2 T}{\partial z^2}\right]+q(r,\ \varphi,\ z,\ t)=\frac{\partial T}{\partial t} \tag{1}$$

with the boundary conditions

$$\left.\frac{\partial T}{\partial r}\right|_{r=a}=hT|_{r=a};\quad \left.\frac{\partial T}{\partial z}\right|_{z=\pm b}=\pm hT\Big|_{z=\pm b}. \tag{2}$$

Taking the Laplace transform of Eq. (1) with respect to time for zero initial conditions and the integral transform

$$\tilde{T}_{1,2}=\frac{1}{2\pi b}\int_0^{2\pi}\int_{-b}^{b}\overline{T}(r,\ \varphi,\ z,\ \lambda)(\sin sz+\cos sz)\left\{\begin{matrix}\cos m\varphi\\ \sin m\varphi\end{matrix}\right\}dz d\varphi, \tag{3}$$

where

$$\overline{T}(r,\ \varphi,\ z,\ \lambda)=\int_0^{\infty}T(r,\ \varphi,\ z,\ t)\exp(-\lambda t)\,dt \tag{4}$$

and s satisfies the equation

$$\operatorname{tg} 2sb=\frac{2hs}{h^2-s^2}, \tag{5}$$

reduces (1) to the form

$$\frac{\partial^2\tilde{T}_{1,2}}{\partial r^2}+\frac{1}{r}\frac{\partial\tilde{T}_{1,2}}{\partial r}-\left[\frac{m^2}{r^2}+s_\mu^2+\frac{\lambda}{a_0^2}\right]\tilde{T}_{1,2}=-\frac{\tilde{q}_{1,2}}{a_0^2}. \tag{6}$$

By solving this equation and taking inverse transforms we find the following expression for the temperature in a finite cylinder:

$$T(r, \varphi, z, t) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty}\sum_{\mu}\left\{\frac{1}{\pi}\sum_{2m}\left[2\sum_{\alpha_n}\frac{\alpha_n^2}{h^2a^2+\alpha^2a^2-m^2}\,\frac{I_m(\alpha_n r)}{I_m^2(\alpha_n a)}\,\frac{\hat{q}_1}{a_0^2\left(\alpha_n^2+s_\mu^2+\frac{\lambda}{a_0^2}\right)}\right]\cos m\varphi + \right.$$

$$\left. + \frac{1}{\pi}\sum_{2m-1}\left[2\sum_{\alpha_n}\frac{\alpha_n^2}{h^2a^2+\alpha^2a^2-m^2}\,\frac{\hat{q}_2}{a_0^2\left(\alpha_n^2+s_\mu^2+\frac{\hat{\lambda}}{a_0^2}\right)}\,\frac{I_m(\alpha_n r)}{I_m^2(\alpha_n a)}\right]\sin m\varphi\right\}\frac{1}{2b}(\sin s_\mu z+\cos s_\mu z)\exp(\lambda t)\Big\}\,d\lambda,$$

where α_n are the roots of the equation

$$\alpha I_m'(\alpha a) + hI_m(\alpha a) = 0, \quad m = 0, 1, 2, \ldots$$

Reg. No. 2093-79 Dep.
Original article submitted Jan. 2, 1978.
Abstract submitted Apr. 19, 1979.

CALCULATION OF TEMPERATURE DISTRIBUTIONS IN HEAT-TRANSFER PROBLEMS

I. N. Bogaenko and M. V. Boichuk

UDC 536.24:681.3

A method is presented for solving thermophysics problems

$$c\rho\frac{\partial}{\partial\tau}\vartheta = \lambda\Delta\vartheta - \sum_{i=1}^{j} v_i(\tau, x)\frac{\partial}{\partial x_i}\vartheta + q, \quad \vartheta = \vartheta_0, \quad \tau = 0, \tag{1}$$

$$\lambda\frac{\partial}{\partial\nu}\vartheta + \alpha(\vartheta - \vartheta_m) = 0, \quad x \in \partial G,$$

where the symbols have the following meanings: c, specific heat; ρ, density; λ, thermal conductivity; $\{v_i(\tau, x\}_{i=1}^{j}$, velocity field of moving substance; τ, time variable; $x = (x_1, \ldots, x_j)$, spatial variable; q, strength of internal heat sources (sinks); ϑ_0, initial temperature; ϑ_m, temperature of medium; ν, inward normal to surface ∂G; α, heat-transfer coefficient.

The method consists in the reduction of boundary-value problem (1) to a system of Volterra integral equations of the second kind using the method of separation and based on the possibility of expanding an arbitrary function in a set of eigenfunctions of the Laplacian operator. The question of improving the convergence of the series describing the solution is examined.

Calculations of temperature distributions in both classical and nonclassical model problems are presented.

Reg. No. 2098-79 Dep.
Original article submitted Dec. 11, 1978.
Abstract submitted Apr. 16, 1979.